Proton-neutron multiplets in exotic ^{134}Sb: testing the shell-model effective interaction

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The experimental level structure of ^{134}Sb is compared with the results of a shell-model calculation in which the two-body matrix elements of the effective interaction have been derived from the CD-Bonn nucleon-nucleon potential. The experimental data, including the very low-lying first-excited 1^- state, are remarkably well reproduced by the theory. The results of this paper complement those of our previous studies on ^{135}Sb and ^{134}Sn, showing that our effective interaction is well suited to describe ^{132}Sn neighbors beyond $N = 82$. The various terms which contribute to the matrix elements of the proton-neutron effective interaction are examined and their relative importance is evidenced.

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The study of nuclei in the regions of shell closures off stability is currently the subject of great interest, with particular attention focused on the neighbors of doubly magic ^{132}Sn. A considerable effort has been recently made to gain information on neutron-rich nuclei beyond the $N = 82$ shell closure. The data which are becoming available appear to be somewhat different from what one might expect by extrapolating the existing results for $N < 82$ nuclei. For instance, the first 2^+ state in ^{134}Sn lies at 726 keV excitation energy, which makes it the lowest first-excited 2^+ level observed in a semi-magic even-even nucleus over the whole chart of nuclides. A significant drop in the energy of the lowest-lying $5/2^+$ state in ^{135}Sb has been observed as compared to the values measured for the Sb isotopes with $N \leq 82$. In ^{134}Sb, the 0^- ground state and the first excited 1^- state are nearly degenerate, the latter lying at 13 keV. This situation is similar to what occurs in ^{210}Bi, which is the counterpart of ^{134}Sb in the lead region. In this case, the lowest multiplet, $\pi h_{9/2}\nu g_{9/2}$, shows a breakdown of the Nordheim strong rule [1] in that the 1^- state becomes the ground state with the 0^- state at about 50 keV excitation energy.

These new data might be seen as the onset of a modification in the shell structure, which, starting at $N = 83 - 84$, is expected to produce more evident effects for larger neutron excess. As an example, a possible explanation of the position of the $5/2^+$ state in ^{135}Sb may reside in a downshift of the $d_{5/2}$ proton level relative to the $g_{7/2}$ one caused by a more diffuse nuclear surface produced by the two neutrons beyond the 82 shell closure. A shell-model calculation using experimental single-particle (SP) energies with the above downshift set at 300 keV and a two-body effective interaction derived from a modern nucleon-nucleon (NN) potential leads indeed to a good description of ^{135}Sb [2, 3]. However, while this Hamiltonian also provides a satisfactory agreement with experiment for ^{134}Sn [2] this is not the case for ^{134}Sb [4]. Other shell-model calculations have been performed for these three nuclei [5, 6, 7], making use of effective interactions obtained through various different modifications of an interaction [8] which was originally constructed for the ^{208}Pb region starting from the Kuo-Herling matrix elements [9]. None of these calculations, however, is able to account simultaneously for the peculiar features of ^{134}Sn, ^{134}Sb, and ^{135}Sb.

In this context, therefore, it is quite a relevant question whether there is a unique consistent Hamiltonian able to do this and, in particular, whether it may be derived from a realistic free NN potential. More precisely, using SP energies taken from the experiment and a two-body effective interaction derived from a modern NN potential, we would like to see how few-valence-particle nuclei just above ^{132}Sn are described so as to ascertain whether effects beyond the standard shell model are really indispensable.

Very recently we have studied ^{135}Sb [10] and ^{134}Sn [11] within the framework of the shell model assuming ^{132}Sn as a closed core and taking as model space for the valence proton and neutrons the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $0h_{11/2}$ of the 50-82 shell and the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the 82-126 shell, respectively. The two-body effective interaction has been derived by means of a $\hat{Q}$-box folded-diagrams method [12] from the CD-Bonn NN potential [13], the short-range repulsion of the latter being renormalized by use of the low-momentum potential $V_{\rm low-k}$ [14]. For protons, the Coulomb interaction has been added to $V_{\rm low-k}$. As for the proton and neutron SP energies, they have been taken from the experimental spectra of ^{133}Sb and ^{133}Sn, respectively, with the exception of the proton $s_{1/2}$ and the neutron $i_{13/2}$ levels which are still missing. The values of $\epsilon_{s_{1/2}}$ and $\epsilon_{i_{13/2}}$ have been taken from Refs. [15] and [16], respectively, where it is discussed how they are determined. All the adopted values of the SP energies

are reported in [10], to which we also refer for a brief discussion of our derivation of the two-body effective interaction.

Here we report on our study of ^{134}Sb, which has been performed using the same Hamiltonian as above. The calculations have been carried out by using the OXBASH shell-model code [17].

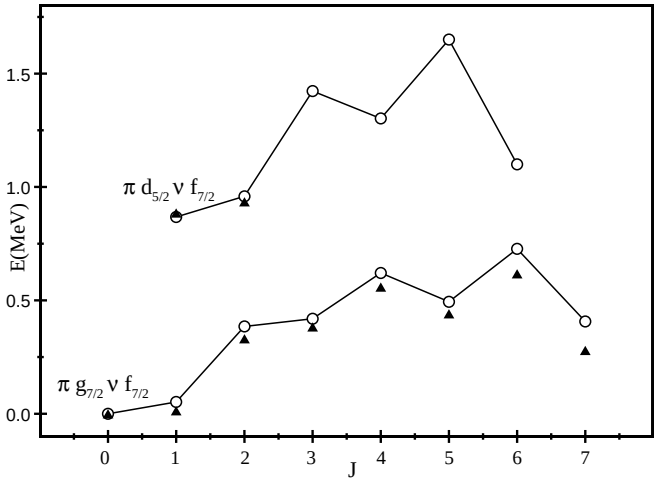

FIG. 1: Proton-neutron $\pi g_{7/2}\nu f_{7/2}$ and $\pi d_{5/2}\nu f_{7/2}$ multiplets in ^{134}Sb. The theoretical results are represented by open circles while the experimental data by solid triangles.

We now present our results starting with the binding energy of the ground state. Our calculated value is 12.85 ± 0.05 MeV, which compares very well with the experimental one, 12.96 ± 0.05 MeV [18]. Note that the error on the calculated value arises from the experimental errors on the proton and neutron separation energies of ^{133}Sb and ^{133}Sn [19].

The calculated energies of the $\pi g_{7/2}\nu f_{7/2}$ and $\pi d_{5/2}\nu f_{7/2}$ multiplets in ^{134}Sb are reported in Fig. 1, where they are compared with the experimental data [4, 20]. The first eight calculated states arise from the $\pi g_{7/2}\nu f_{7/2}$ configuration and have their experimental counterpart in the eight lowest-lying experimental states. The wave functions of these states are characterized by very little configuration mixing, the percentage of the leading component having a minimum value of 88% for the $J^\pi = 2^-$ state while ranging from 94% to 100% for all other states. As for the $\pi d_{5/2}\nu f_{7/2}$ multiplet, we find that the 1^-, 2^-, 4^-, and 6^- members correspond to the yrare states, while both the other two, with $J^\pi = 3^-$ and 5^-, to the third excited state. In fact, the second 3^- and 5^- states, which are predicted at 1.42 and 1.46 MeV excitation energy, belong to the $\pi g_{7/2}\nu p_{3/2}$ configuration. As is shown in Fig. 1, only the 1^- and 2^- members of the $\pi d_{5/2}\nu f_{7/2}$ multiplet are known. Actually, a (5^-) state at 1.38 MeV has been observed which we identify, however, with our second 5^- state. As regards the structure of the states belonging to the $\pi d_{5/2}\nu f_{7/2}$ multiplet, we find that all members receive significant contributions from configurations other than the dominant one, their percentage reaching even 50% in the case of the 2^- state.

We see that the agreement between theory and experiment is very good, the discrepancies being in the order of a few tens of keV for most of the states. The largest discrepancy occurs for the 7^- state, which lies at about 130 keV above its experimental counterpart. It is an important outcome of our calculation that we predict almost the right spacing between the 0^- ground state and first excited 1^- state. In fact, the latter has been observed at 13 keV excitation energy, our value being 53 keV. It is worth mentioning that in the preliminary calculations of Ref. [21] we overestimated the excitation energy of the 1^- state by about 200 keV. Comments on this point can be found in Ref. [10].

In the recent paper mentioned above, Ref. [4], calculations have been performed for ^{134}Sb using an effective interaction derived from the CD-Bonn-96 NN potential by means of a G-matrix folded-diagram method including diagrams up to third order [22]. From comparison with experiment for the eight states of the $\pi g_{7/2}\nu f_{7/2}$ multiplet, an average deviation of about 150 keV was found, and, in particular, the 1^- state turned out to lie at 329 keV above the 0^- ground state. As for the $\pi d_{5/2}\nu f_{7/2}$ multiplet, while the 2^- state is overestimated by about 120 keV the discrepancy for the 1^- state reaches about 380 keV. In the same paper, results are also presented which have been obtained with different two-body matrix elements, namely those of the KH208 [8] and KH5082 interactions [5]. Calculations for ^{134}Sb with the latter interaction were already performed in Ref. [7]. The authors of [4] present the results of both calculations pointing out that they are in better agreement with the experimental data than those obtained with the CD-Bonn interaction. On the other hand, they also draw attention on the fact that for ^{135}Sb the latter interaction does better than the KH208 interaction (see Ref. [3] for details). On these grounds, their conclusion [4] is that a consistent Hamiltonian for the three nuclei ^{134}Sn, ^{134}Sb, and ^{135}Sb has yet to be found.

Our point of view is quite different. In fact, we have shown here that our realistic effective interaction gives a good description of ^{134}Sb while the results presented in Refs. [10, 11] evidence that it is also able to reproduce the spectroscopic properties of ^{135}Sb and ^{134}Sn. In particular, in [10] we have reported the diagonal matrix elements of the proton-neutron effective interaction for the $\pi g_{7/2}\nu f_{7/2}$ and $\pi d_{5/2}\nu f_{7/2}$ configurations and pointed out their crucial role for the structure of the low-lying $5/2^+$ state in ^{135}Sb. As for ^{134}Sn, it is worth noting that the results obtained in [11] are not significantly different from those of [16, 23], where slightly different effective interactions were employed.

In this context, it is interesting to try to understand

what makes our proton-neutron matrix elements appropriate to the description of the multiplets in ^{134}Sb, in particular the very small energy spacing between the 0^- and the 1^- states. To this end, we have performed an analysis of the various contributions to our effective interaction, focusing attention on the $\pi g_{7/2}\nu f_{7/2}$ configuration. In fact, in this case a direct relation can be established between the diagonal matrix elements and the behavior of the multiplet, since its members are, as mentioned above, almost pure. We start by reporting in Fig. 2 the behavior of the diagonal matrix elements of the effective interaction as a a function of J. We see, as expected, that the pattern is quite similar to that of the corresponding multiplet in Fig. 1 and the matrix elements for the 0^- and the 1^- states are almost equal.

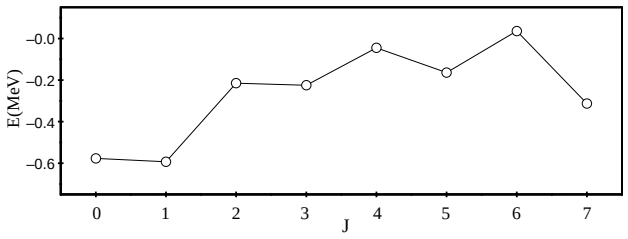

FIG. 2: Diagonal matrix elements of the two-body effective interaction for the $\pi g_{7/2}\nu f_{7/2}$ configuration.

As mentioned above, our effective interaction is calculated within the framework of a $\hat{Q}$-box folded-diagram method. In particular, the $\hat{Q}$ box is composed of first- and second-order diagrams in the $V_{\rm low-k}$ derived from the CD-Bonn potential. In other words, the matrix elements of the effective interaction contain the $V_{\rm low-k}$ plus additional terms which take into account core-polarization effects arising from $1p-1h$ ("bubble" diagram) and $2p-2h$ excitations. They also include the so-called ladder diagrams, which must compensate for the excluded configurations above the chosen model space. We may point out that the effective interaction is obtained by summing the $\hat{Q}$-box folded-diagram series [24].

In Fig. 3 we show the $\pi g_{7/2}\nu f_{7/2}$ matrix elements of the $V_{\rm low-k}$ as a function of J together with the second-order two-body contributions. From the inspection of this figure we see that the incorrect behavior of the $V_{\rm low-k}$ matrix elements is "healed" by the V_{1p1h}, V_{2p2h} and $V_{\rm ladder}$ corrections. In particular it appears that a crucial role is played by the bubble diagram, especially as regards the position of the 1^- state. In this connection, we may mention that the effect of the bubble renormalization on the $0^- - 1^-$ splitting was also pointed out in Ref. [4] with regard to the Kuo-Herling interaction. It is worth noting that in our calculations the folding procedure turns out to provide a common attenuation of all matrix elements, which does not affect the overall behavior of the multiplet.

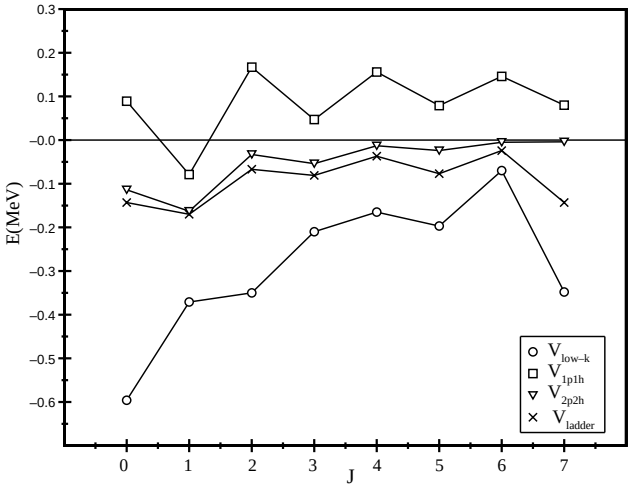

FIG. 3: Diagonal matrix elements of $V_{\rm low-k}$ and contributions from the two-body second-order diagrams for the $\pi g_{7/2}\nu f_{7/2}$ configuration. See text for comments.

To summarize, we have shown that our realistic shell-model calculation for ^{134}Sb leads to a very good agreement with experiment for the $\pi g_{7/2}\nu f_{7/2}$ multiplet as well as for the two observed members of the $\pi d_{5/2}\nu f_{7/2}$ multiplet. This outcome, when considered along with the results we have obtained for ^{134}Sn and ^{135}Sb, evidences the merit of our effective interaction in describing the properties of ^{132}Sn neighbors with neutrons beyond the 82 shell. We have also examined the various terms of our effective interaction, in order to understand their relative importance in the resulting final matrix elements. We have evidenced the importance of the renormalizations one has to introduce to account for the limited size of the chosen model space, in particular those arising from the $1p-1h$ excitations. Based on these results, we may conclude that to explain the presently available data on neutron-rich nuclei beyond ^{132}Sn there is no need to invoke shell-structure modifications.

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[1] L. W. Nordheim, Phys Rev. **78**, 294 (1950); Rev. Mod Phys. **23**, 322 (1951).
[2] J. Shergur *et al.*, Phys. Rev. C **65**, 034313 (2002).
[3] J. Shergur *et al.*, Phys. Rev. C **72**, 024305 (2005).
[4] J. Shergur *et al.*, Phys. Rev. C **71**, 064321 (2005).
[5] W. T. Chou and E. K. Warburton, Phys. Rev. C **45**, 1720 (1992).
[6] S. Sarkar and M. S. Sarkar, Phys. Rev. C **64**, 014312 (12001).
[7] A. Korgul *et al.*, Eur. Phys. J. A **15**, 181 (2002).
[8] E. K. Warburton and B. A. Brown, Phys. Rev. C **43**, 602 (1991).

[9] T. T. S. Kuo and G. H. Herling, US Naval Research Laboratory Report No. 2258, 1971 (unpublished).
[10] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C, **72** 057302 (2005).
[11] A. Covello, L. Coraggio, A. Gargano, and N. Itaco, BgNS Transactions, in press.
[12] T. T. S. Kuo and E. Osnes, *Lecture Notes in Physics*, Vol. 364, (Springer-Verlag, Berlin, 1990).
[13] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[14] S. Bogner, T. T. S. Kuo, L. Coraggio, A. Covello, and N. Itaco, Phys. Rev. C **65**, 051301 (2002).
[15] F. Andreozzi, L. Coraggio, A. Covello, A. Gargano, T. T. S. Kuo, and A. Porrino, Phys. Rev. C **56**, R16 (1997).
[16] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **65**, 051306(R) (2002).
[17] B. A. Brown, A. Etchegoyen, and W. D. M. Rae, The computer code OXBAH, MSU-NSCL, Report No. 534.
[18] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[19] B. Fogelberg, K. A. Mezilev, H. Mach, V. I. Isakov, and J. Slivova, Phys. Rev. Lett. **82**, 1823 (1999).
[20] Data extracted using the NNDC On-line Data Service from the XUNDL database, file revised as of November 8, 2005.
[21] A. Gargano, Eur. Phys. J. A **20**, 103 (2004).
[22] B. A. Brown, N. J. Stone, J. R. Stone, L. S. Towner, and M. Hjorth-Jensen, Phys Rev, C **71**, 044317 (2005) and **72**, 029901(E) (2005).
[23] A. Covello, L. Coraggio, A. Gargano, and N. Itaco, Journal of Physics: Conference Series **20**, 137 (2005).
[24] J. Shurpin, T. T. S. Kuo, D. Stottman, Nucl. Phys. A **408**, 310 (1983).